Abstract:

An algorithm for determining the marginal queue length probability distributions for a closed queueing network is presented. Efficient algorithms for determining the normalizing constant are known. The marginal distributions are obtained at minimal additional cost. Further, the algorithm does not depend on the product form solution. Thus it may be applied to closed queueing networks with classes of customers. It is shown how the algorithm may be used to find the queue length probability distribution for each class in such a network.

A Note on the Solution of General Closed Queueing Networks

By

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Note that due to the nature of the transitions of certain classes we may have \( f_m(k) = 0 \) for \( k \) greater than some number of customers.

Classes of customers may be employed in several ways in network models. When they are used in an artificial way the marginal queue length probability distributions for the individual classes have no meaning. For example, classes may be used to give customers different transition probabilities on departure from server \( i \) depending on whether they had come from server \( j \) or server \( k \). In such cases the summation (8) may be solved using Horner's rule. This may be used to sum either \( g_m(y_m) \) as given in (6) or (7) since the \( n_m! \) of (7) can be brought outside the summation. The algorithm presented here may then be used to find \( G(M,N) \) and the \( H_m(M,N-k) \).

In other instances the marginal queue length probability distributions of the individual classes may be of interest. For instance algorithms to balance the loading between terminal and batch jobs may be investigated in this way. To find these probability distributions we proceed as follows. Evaluate the summation (8) using the algorithm given above with each row representing a class. If this is done for each server we have \( M \) arrays \( Z_m \) for \( m = 1, 2, \ldots, M \). The interpretation of these arrays is:

1. If \( g_m(y_m) \) is given by (6) then \( f_m(n_m) = Z_m(R+1,n_m) \).
   If \( g_m(y_m) \) is given by (7) then \( f_m(n_m) = n_m!Z_m(R+1,n_m) \). In either case \( n_m = 0, 1, \ldots, N \).

2. The elements \( Z_m(r,n_m-k) \) for \( r = 1, 2, \ldots, R \) and \( k = 0, 1, \ldots, n_m \) are analogous to the functions \( H_m(M,N-k) \).

This additional step will require \( O(M^2R^2) \) operations.
The probability of \( j \) customers of class \( r \) at server \( m \) conditioned on the presence of \( k \) customers of all classes is:

\[
P(n_{mr} = j | n_m = k) = h_{mr}(j)Z_m(r, k-j)/Z_m(R+1, k)
\]  \hspace{1cm} (9)

The derivation of (9) follows that of (5). Using the values obtained for \( f_m(n_m) \) we may proceed to find the queue length probability distribution for each server. To find the queue length distribution for a particular class we compute

\[
P(n_{mr} = j) = \sum_{k=j}^{N} P(n_m = k)P(n_{mr} = j | n_m = k)
\]

\[
= \left( h_{mr}(j)/G(M, N) \right) \sum_{k=j}^{N} Z(m, N-k)Z_m(r, k-j)
\]

We have presented an algorithm of \( O(M^2N^2) \) for determining the marginal queue length probability distributions of a closed queueing network. The importance of this algorithm arises when the functions \( f_m(n_m) \) do not have the form of (2). This includes the case of closed networks with more than one class of customer and either PCFS, processor sharing or LCFS scheduling rules. A method for determining the queue length probability distributions for the individual classes was also presented. In this case the marginal queue length probability distributions for each class may be determined in \( O(M^2N^2R^2) \) operations.
Bibliography


